ΑΡΙΣΤΟΤΕΛΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΟΝΙΚΗΣ

ΤΜΗΜΑ ΧΗΜΙΚΩΝ ΜΗΧΑΝΙΚΩΝ
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PROCESS DYNAMICS & CONTROL **2014**

Final Review Series of Solved Problems

Problem 1:

Consider the following feedback control system of Figure 1.

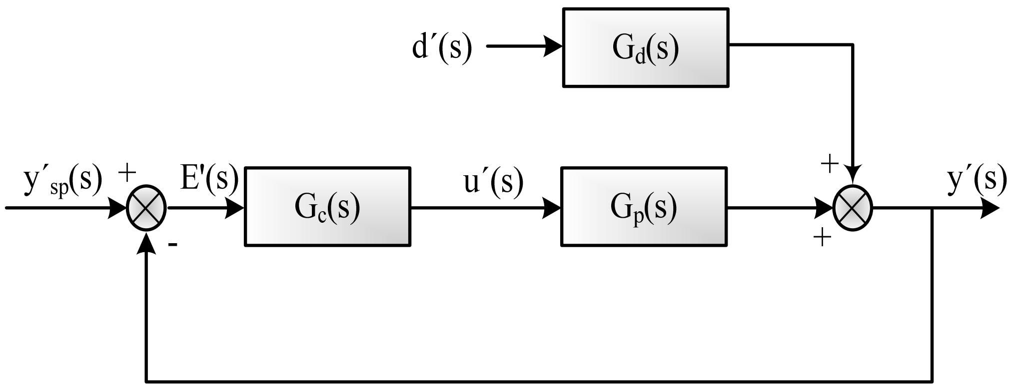

Figure 1: Feedback control-loop.

The process and disturbance transfer functions are assumed to be equal.

$$G_p(s) = G_d(s) = \frac{K_p}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)(\tau_4 s+1)}$$

$$\tau_1 = 1 \text{ min} \quad ; \quad \tau_2 = 2 \text{ min} \quad ; \quad \tau_3 = 4 \text{ min} \quad ; \quad \tau_4 = 7 \text{ min} \quad ; \quad K_p = 1$$

i) Calculate the Ziegler-Nichols settings (K_c, τ_I and τ_D) of a PID controller based on the continuous cycling method using the characteristic equation of the closed-loop system.

ii) Compare the calculated values of K_{cu} and P_u (see question (i)) with the results of Figure 2.

iii) Calculate an approximate FOPTD model of the process using the experimental results of Figure 3.

iv) Use the identified FOPTD model to determine the Cohen-Coon and ITAE settings (K_c, τ_I and τ_D) for a PID controller.

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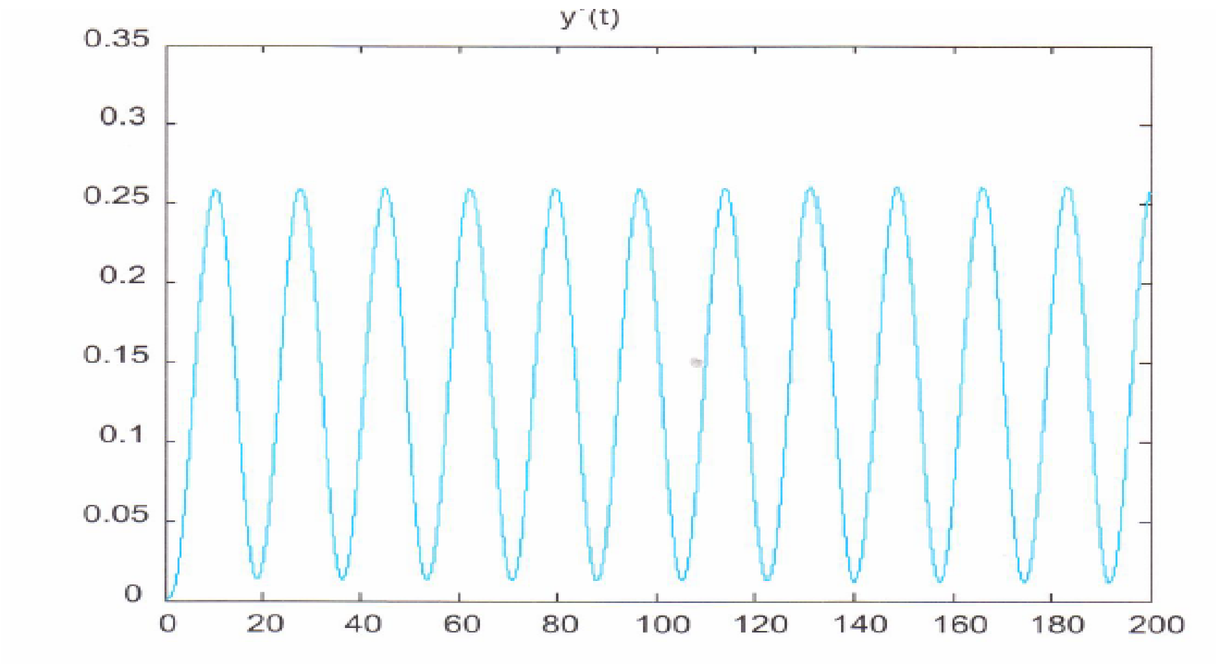

Figure 2: Experimental calculation of the ultimate gain, K_{cu}= 6.35 and of the ultimate period, P_u.

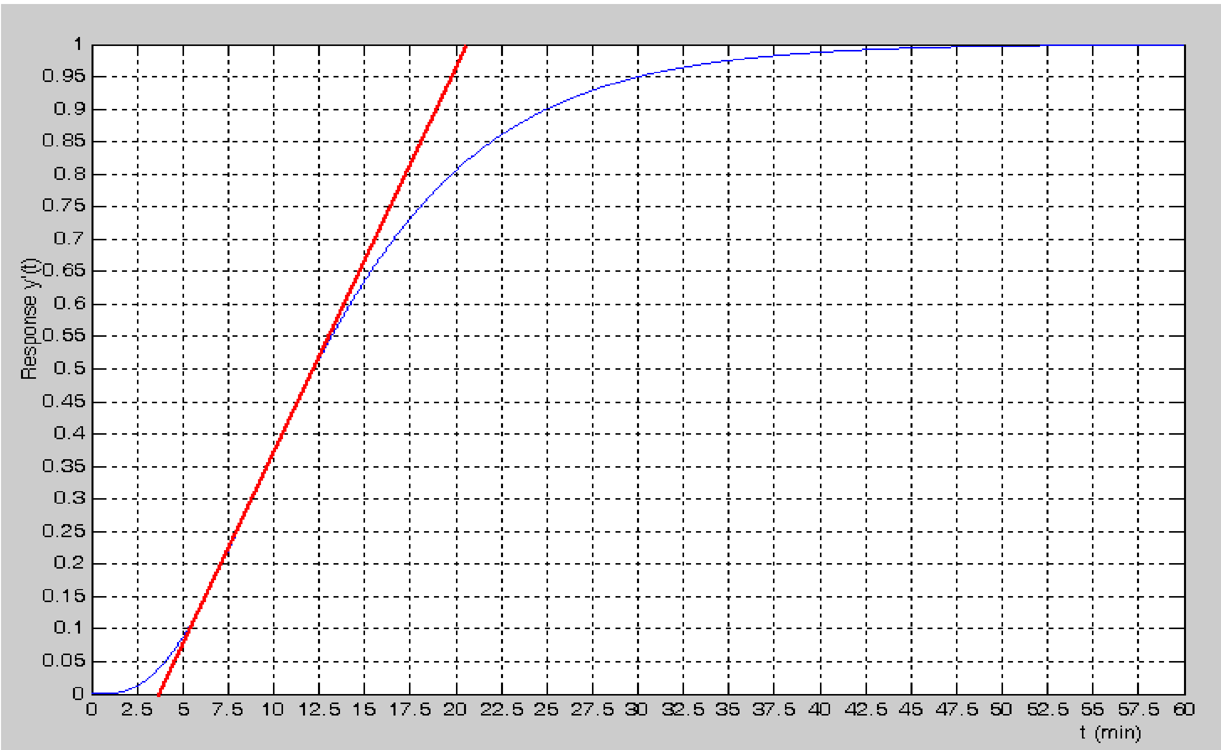

Figure 3: Open-loop response to a unit step change in the input signal (Process reaction curve)

Solution of Problem 1:

i) The characteristic equation of the closed-loop system under P-control is:

$$1+G_{OL}(s)=0 \Rightarrow 1+\frac{K_c}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)(\tau_4 s+1)}=1+\frac{K_c}{(s+1)(2s+1)(4s+1)(7s+1)}=0$$

$$\boxed{56s^4+106s^3+63s^2+14s+(1+K_c)=0}$$

To calculate K_{cu} and ω_c, we substitute s = jω in the above characteristic equation. Thus, we have:

$$56\omega^4 - 106\omega^3 j - 63\omega^2 + 14j\omega + (1+K_c) = 0$$

Rearranging the above equation in the complex form (**Re** + j**Im**) we obtain:

$$\boxed{(1+K_c+56\omega^4-63\omega^2)+j(-106\omega^3+14\omega)=0}$$

This complex equation is satisfied if both the real and imaginary parts are equal to zero.

Real part (Re): $(1+K_c+56\omega^4-63\omega^2)=0$ (a)

Imaginary part (Im): $(-106\omega^3+14\omega)j=-j\omega(106\omega^2-14)=0$ (b)

From Eq. (b), we get: $\omega^2=14/106$ and thus, $\omega_c=0.3634$.

Substituting the value of ω into Eq. (a), we calculate the ultimate value of K_{cu}.

$$1+K_{cu}+56\cdot 0.3634^4-63\cdot 0.3634^2=0 \Rightarrow \boxed{K_{cu}=6.3439}$$

Accordingly, $P_u=\frac{2\pi}{\omega}=\frac{2\cdot 3.14}{0.3634} \Rightarrow \boxed{P_u=17.28}$

From the above results we can calculate the following Z-N tuning settings of a PID controller:

- PID: $K_c = 0.6K_{cu} = 3.8063$; $\tau_I = P_u/2 = 8.64$ and $\tau_D = P_u/8 = 2.16$

ii) From Figure 2, we can verify the above calculated values of the ultimate gain and ultimate period are:

$$\boxed{K_{cu}=6.35\approx 6.3439} \quad \text{and} \quad \boxed{P_u=17.4\approx 17.28}$$

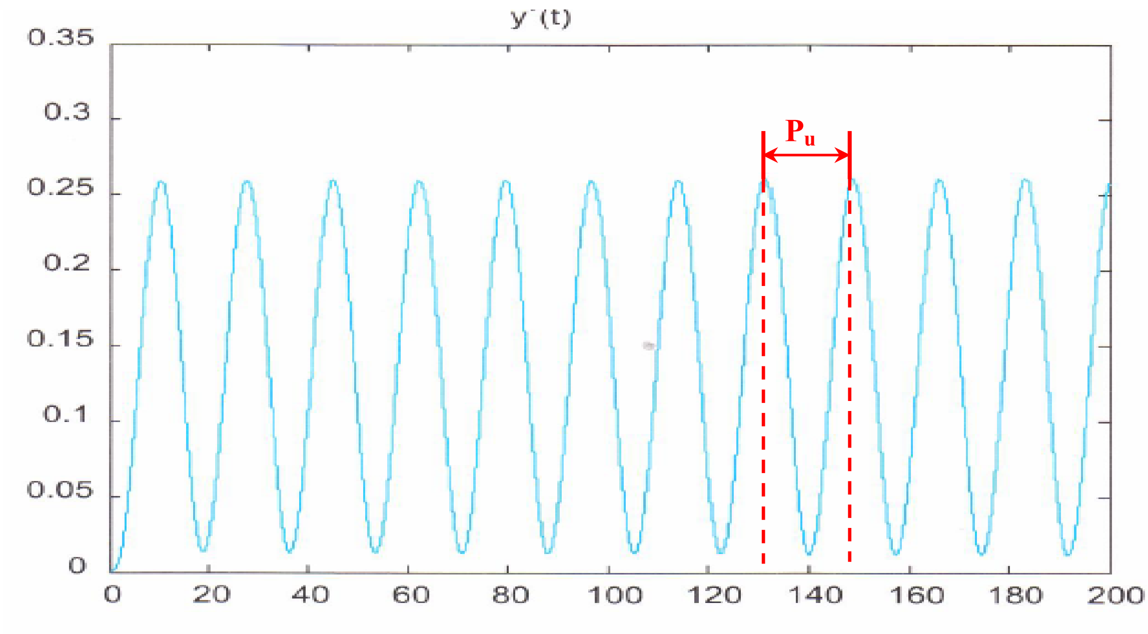

Figure 2: Experimental calculation of the ultimate gain, K_{cu}= 6.35 and of the ultimate period, P_u.

ii) From the results of Fig. 3 we can obtain the following parameter values for a FOPTD model.

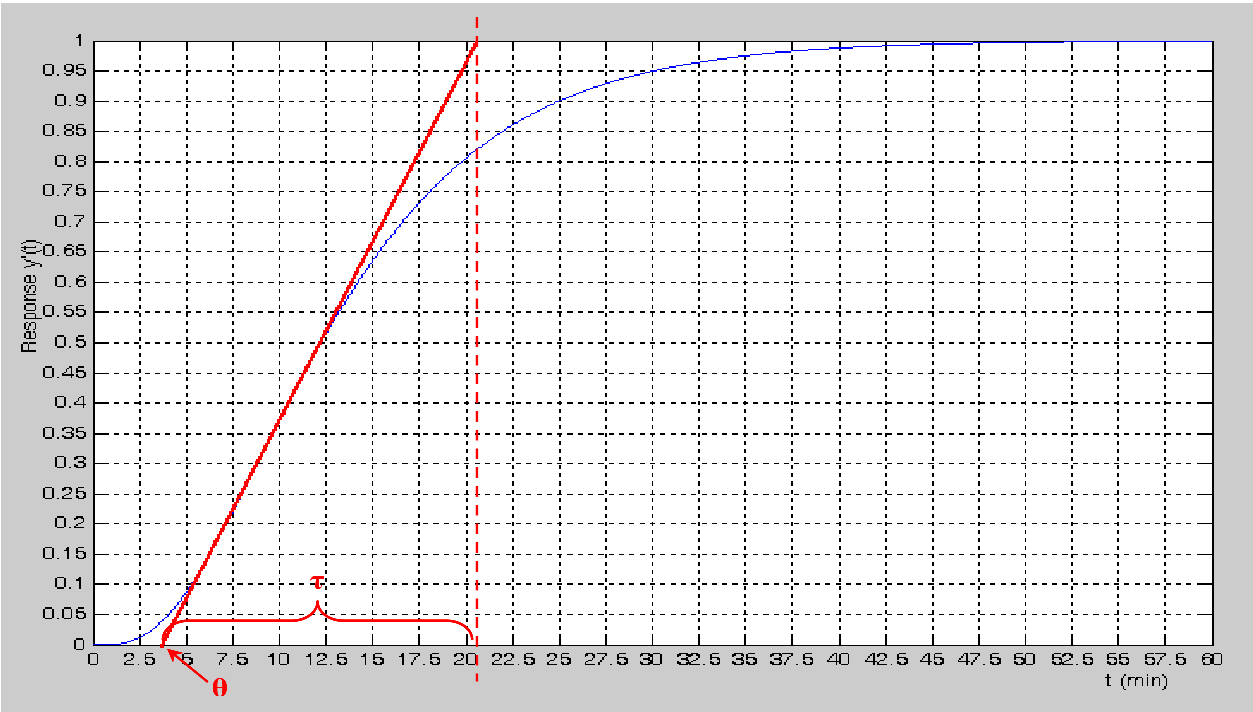

Figure 3: Open-loop response to a unit step change in the input signal (Process Reaction Curve)

$K = 1$; $\tau = 16.9$; $\theta = 3.85$

Therefore, the FOPTD model of the process will be:

$$G(s) = \frac{(1)\cdot e^{-3.85s}}{(16.9s+1)}$$

iv) From the attached two Tables 1 and 2, we can calculate the **Cohen-Coon and ITAE settings** (K_c, τ_I and τ_D) of a PID controller.

Cohen-Coon settings

$$K_c = \frac{1}{K}\frac{\tau}{\theta}\left(\frac{3\theta+16\tau}{12\tau}\right) = 6.1028 \quad ; \quad \tau_I = \frac{\theta[32+6(\theta/\tau)]}{13+8(\theta/\tau)} = 8.6667 \quad ;$$

$$\tau_D = \frac{4\theta}{11+2(\theta/\tau)} = 1.3443$$

ITAE settings

Set-point Settings

$$KK_c = 0.965(3.85/16.9)^{-0.85} = 3.3930 \Rightarrow K_c = 3.3930/1 \Rightarrow K_c = 3.3930$$

$$\tau/\tau_I = 0.796 - 0.1465(3.85/16.9) = 0.7626 \Rightarrow \tau_I = 16.9/0.7626 \Rightarrow \tau_I = 22.1603$$

$$\tau_D/\tau = 0.308(3.85/16.9)^{0.929} = 0.0779 \Rightarrow \tau_D = 0.0779\cdot 16.9 \Rightarrow \tau_D = 1.3171$$

Load Settings

$$KK_c = 1.357(3.85/16.9)^{-0.947} = 5.5075 \Rightarrow K_c = 5.5075/1 \Rightarrow K_c = 5.5075$$

$$\tau/\tau_I = 0.842(3.85/16.9)^{-0.738} = 2.5085 \Rightarrow \tau_I = 16.9/2.5085 \Rightarrow \tau_I = 6.7370$$

$$\tau_D/\tau = 0.381(3.85/16.9)^{0.995} = 0.0874 \Rightarrow \tau_D = 0.0874\cdot 16.9 \Rightarrow \tau_D = 1.4777$$

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Table 1: Cohen-Coon Tuning Relations

Controller	K_c	T_i	T_d
P-only	$(1/K_p)(\tau/\theta)[1+\theta/3\tau]$		
PI	$(1/K_p)(\tau/\theta)[0.9+\theta/12\tau]$	$\dfrac{\theta[30+3(\theta/\tau)]}{9+20(\theta/\tau)}$	
PID	$(1/K_p)(\tau/\theta)[\dfrac{3\theta+16\tau}{12\tau}]$	$\dfrac{\theta[32+6(\theta/\tau)]}{13+8(\theta/\tau)}$	$\dfrac{4\theta}{11+2(\theta/\tau)}$

Table 2: Controller Design Based on ITAE

Type of Input	Type of Controller	Mode	A	B
Disturbance	PI	P	0.859	−0.977
		I	0.674	−0.680
Disturbance	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a] Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.

[b] For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$.

Problem 2:

Consider the problem of controlling the level in tank 3 of the three tank process shown in Figure 4.

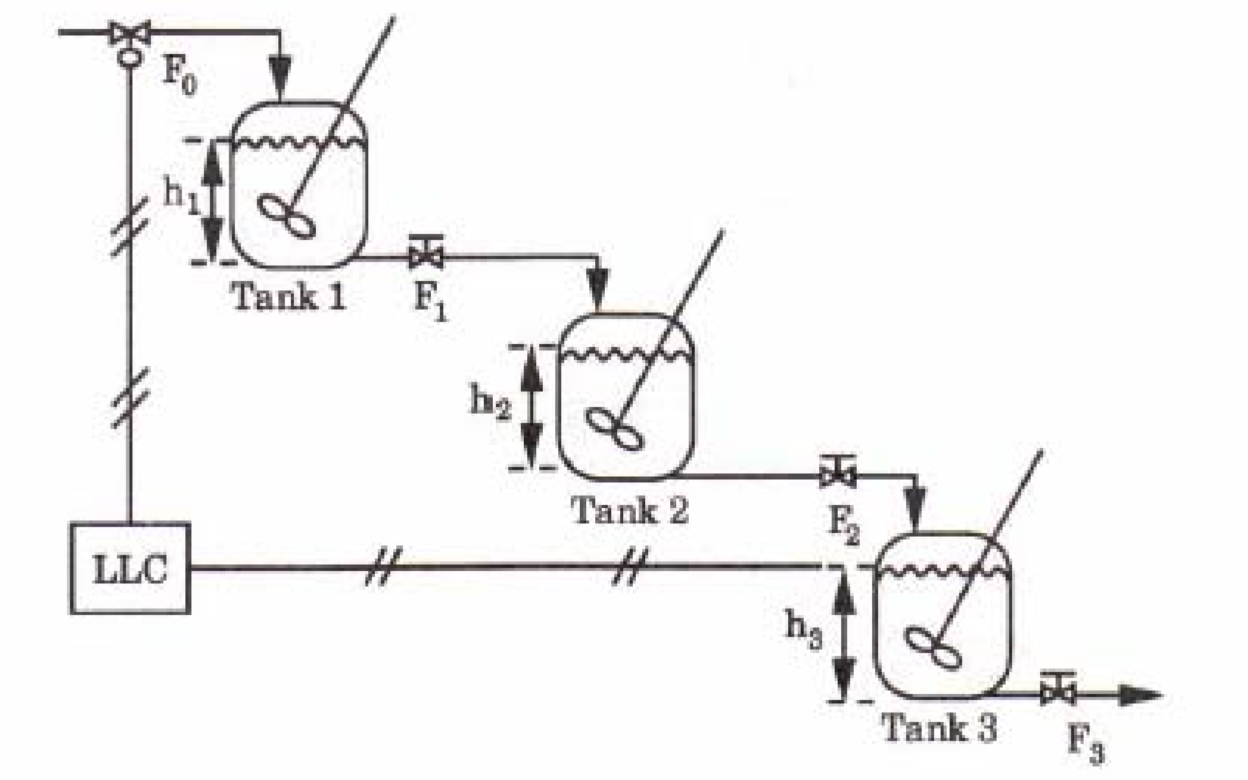

Figure 4: Three noninteracting tanks in series with proportional feedback control

To accomplish this, the inlet flowrate to the first tank, $F_o(t)$, is manipulated based on a signal from a proportonal feedback controller. The process transfer function for the three noninteracting tanks in series is:

$$G_p(s) = K_p / [(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)]$$

where $K_p = 6$ and $\tau_1 = 2$, $\tau_2 = 4$, $\tau_3 = 6$. ***The time constants are given in minutes.***

The trnsfer functions of the measurement element (level transmitter) and final control element (control valve) are equal to one, i.e. $G_m(s) = G_v(s) = 1$.

i) Draw the block diagram for the above control system.

ii) Identify the transfer function for each element in the control system and calculate the closed loop transfer function between the controlled variable Y(s) and the set-point value, $Y_{sp}(s)$.

iii) Using the Routh array calculate the values of the ultimate gain, K_{cu}, and ultimate period of sustained oscillations, P_u.

iv) Use the method of direct substitution s = jω and the characteristic equation, to verify the results of question (iii).

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v) Calculate the Ziegler-Nichols settings of a PID controller, using the tuning rules of the following Table 3.

Table 3: Ziegler-Nichols controller tuning parameters

Ziegler-Nichols	K_c	τ_I	τ_D
P	$0.5K_{cu}$	—	—
PI	$0.45K_{cu}$	$P_u/1.2$	—
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$

vi) For feedback controlling tuning purposes, the approximate model most commonly employed has a first-order-plus-time-delay (FOPTD) transfer function.

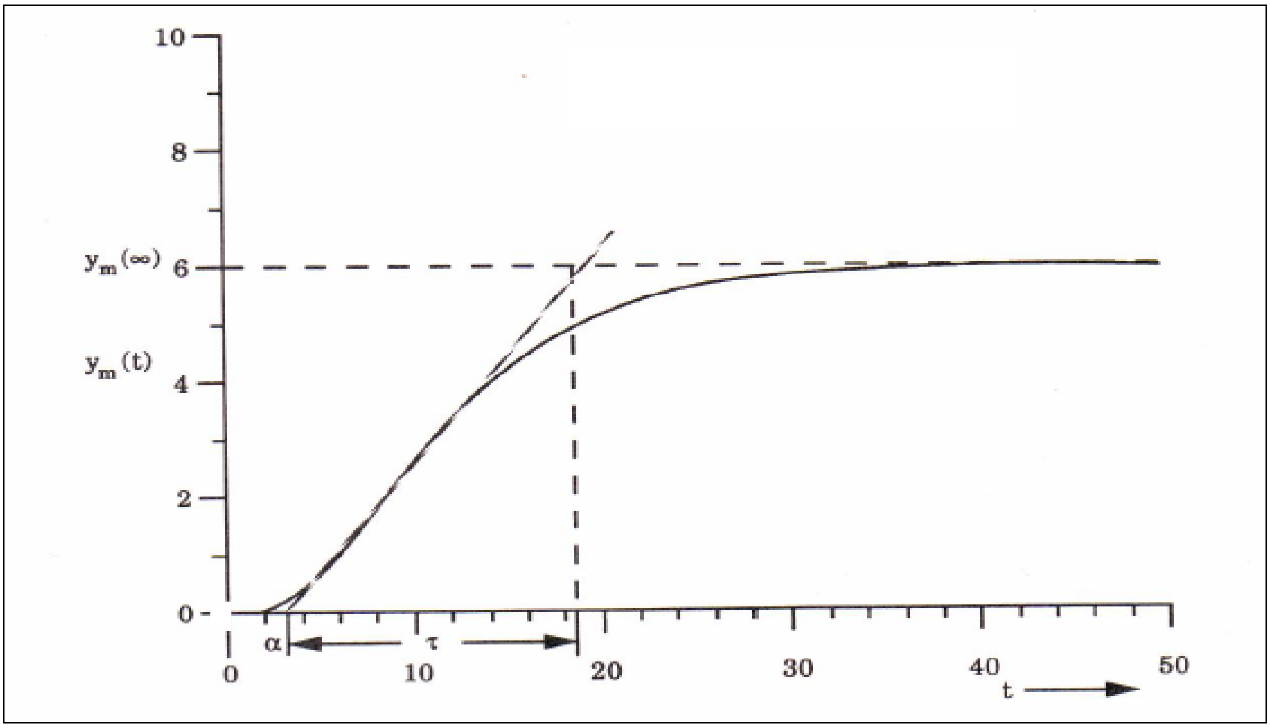


Figure 5: Process reaction curve for the three-tank in series (time is in min)

To estimate the parameters of the approximate model (FOPTD) for the three-tank system, the process reaction curve was applied using a unit step change as a process input. The results of the process reaction curve are given in Figure 5. Applying the graphical procedure, identify the parameters K_p, θ, and τ of the approximate FOPTD process model.

vii) Use the identified FOPTD model to determine the ITAE settings (K_c, τ_I and τ_D) of a PID controller for ***set point changes*** (see Table 4).

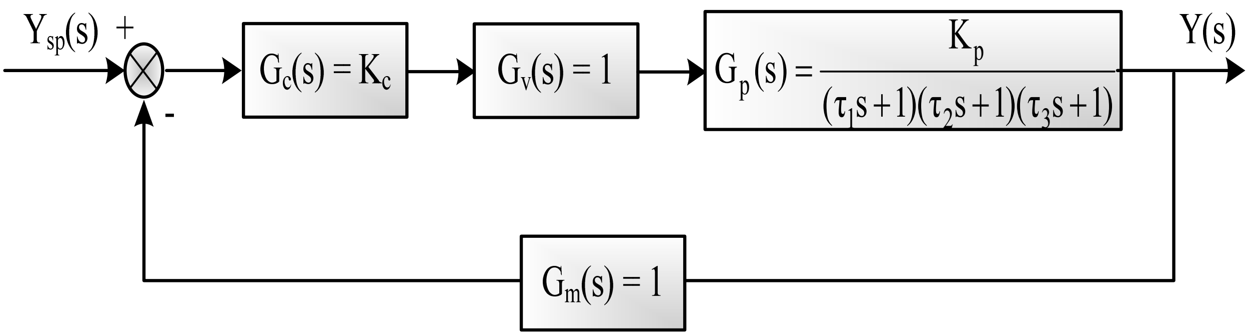
Table 4: Controller Design Based on ITAE and FOPTD Model

Type of Input	Type of Controller	Mode	A	B
Disturbance	PI	P	0.859	−0.977
		I	0.674	−0.680
Disturbance	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a] Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b] For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$.

Solution of Problem 2:

i) The block diagram of the control system is illustrated in the figure below:



ii) Identification of the transfer functions for each element:

TF of measurement device

$$G_m(s) = K_m = 1$$

TF of control valve

$$G_v(s) = K_v = 1$$

TF of P-controller

$$G_c(s) = K_c$$

TF of process

$$G_p(s) = \frac{K_p}{(\tau_1 s+1)(\tau_2 s+1)(\tau_2 s+1)} = \frac{6}{(2s+1)(4s+1)(6s+1)}$$

Derive CLTF:

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_m G_c(s) G_v(s) G_p(s)}{1+G_c(s)G_v(s)G_p(s)G_m(s)} = \frac{\frac{6K_c}{(2s+1)(4s+1)(6s+1)}}{1+\frac{6K_c}{(2s+1)(4s+1)(6s+1)}} \Rightarrow$$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{6K_c}{48s^3+44s^2+12s+(1+6K_c)}$$

iii) The Characteristic Equation is:

$$48s^3+44s^2+12s+(1+6K_c)=0$$

The Routh Array for this third-order system is:

$$\begin{array}{c|cc} 1 & a_n & a_{n-2} \\ 2 & a_{n-1} & a_{n-3} \\ 3 & b_1 & b_2 \\ 4 & c_1 & c_2 \end{array} = \begin{array}{c|cc} 1 & 48 & 12 \\ 2 & 44 & 1+6K_c \\ 3 & 528-48(1+6K_c)/44 & 0 \\ 4 & 1+6K_c & 0 \end{array}$$

To have a stable system, each element of the first column in the Routh array must be positive, i.e.

$$\frac{528-48(1+6K_c)}{44} > 0 \Rightarrow K_c < 5/3 \quad \text{and} \quad 1+6K_c > 0 \Rightarrow K_c > -1/6$$

We conclude that the system will be stable if, $-0.167 < K_c < 1.667$.

Therefore, the controller's gain at the point of marginal stability, i.e. the ultimate gain, is equal to: $K_{cu} = 5/3 = 1.667$.

We can calculate the purely imaginary roots from the following equation:

$$44s^2+(1+6K_{cu})=0 \Rightarrow 44s^2+(1+6\frac{5}{3}) = 44s^2+11=0 \Rightarrow s=\pm j0.5\text{, which yields, } \omega_c = 0.5$$

Hence, $P_u = 2\pi/\omega_c = 2\pi/0.5 = 4\pi \Rightarrow$ $P_u = 12.56$ (in min/cycle).

iv) By substituting $s = j\omega$ into the above characteristic equation, we get:

$$-48j\omega^3 - 44\omega^2 + 12j\omega + (1 + 6K_c) = 0$$

Rearranging the above equation in the complex variable form: (**Re** + j**Im**) we obtain:

$$\boxed{(1 + 6K_c - 44\omega^2) + j(-48\omega^3 + 12\omega) = 0}$$

This complex equation is satisfied if and only if both the real and imaginary parts are equal to zero.

Real part (Re): $(1 + 6K_c - 44\omega^2) = 0$ (a)

Imaginary part (Im): $(-48\omega^3 + 12\omega)j = -j\omega(48\omega^2 - 12) = 0$ (b)

From Eq. (b), we get: $\omega(48\omega^2 - 12) = 0 \Rightarrow \begin{cases} \omega = 0 \\ (48\omega^2 - 12) = 0 \end{cases}$ and thus, $\omega = 0$ or 0.5.

Substituting the values of ω into Eq. (a), we can calculate the K_c, i.e.

For ω = 0 we obtain: $1 + 6K_c = 0 \Rightarrow K_c = -1/6$

For ω = 0.5 we obtain: $1 + 6K_c - 44 \cdot 0.5^2 = 0 \Rightarrow K_c = 5/3$

which confirm the results of question (iii).

v) The Z-N PID controller tuning parameters will be:

$$K_c = 0.6K_{cu} \Rightarrow \boxed{K_c = 1} \; ; \; \tau_I = P_u/2 \Rightarrow \boxed{\tau_I = 6.28 \text{ min}} \; ; \; \tau_D = P_u/8 \Rightarrow \boxed{\tau_D = 1.57 \text{ min}}$$

vi) From Figure 2 we can calculate the numerical values of the FOPTD model parameters:

$$\boxed{K_p = 6} \; ; \; \boxed{\tau = 15 \text{ min}} \; ; \; \boxed{\theta = 3}$$

Therefore, the FOPTD model of the process will be:

$$\boxed{G(s) = \frac{6e^{-3s}}{(15s + 1)}}$$

vii) Finally, from the Table 4 and the above model we get the following ITAE settings of a PID controller for set point changes:

$$KK_c = 0.965(3/15)^{-0.85} = 3.790 \Rightarrow K_c = 3.790/6 \Rightarrow \boxed{K_c = 0.632}$$

$$\tau/\tau_I = 0.796 - 0.1465(3/15) = 0.7667 \;\Rightarrow\; \tau_I = 15/0.7667 \;\Rightarrow\; \boxed{\tau_I = 19.56 \text{ min}}$$

$$\tau_D/\tau = 0.308(3/15)^{0.929} = 0.069 \;\Rightarrow\; \tau_D = 0.069\cdot 15 \;\Rightarrow\; \boxed{\tau_D = 1.04 \text{ min}}$$

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